According to Piaget’s constructivism, children develop their ability to think logico-mathematically by reasoning actively. An earlier Young Children article, “Modifying a Board Game to Foster Kindergartners’ Logico-Mathematical Thinking” (Kamii 2003), shows how the board game Sorry! can be changed to encourage kindergartners to think. The present article is similar but deals with a card game called Lining Up the 5s (see “How to Play Lining Up the 5s,” p. 2), a modified version of Card Dominoes (Kamii & DeVries 1980) (see “How to Play Card Dominoes,” p. 2).

We describe how four groups of children played the game Lining Up the 5s at four different levels. We conclude by arguing that encouraging children to think logico-mathematically is a more valid goal than trying to delineate specific standards for three- to six-year-olds’ mathematics education. (Logico-mathematical thinking will be explained shortly.)

How children played the game

On the following pages, four examples of children playing Lining Up the 5s are not exactly in developmental order, but they generally begin at a lower level of thinking and end at a higher level. Some of the evidence for our interpretations of children’s thinking was based on interviews we conducted with children after we viewed videotaped sessions of the games. Almost all the children in the four examples were six years old.

Yasuhiko Kato, MA, is a professor at Chugoku Gakuen University in Okayama, Japan. He applies to Japanese early education a curriculum based on Piaget’s constructivism, developed by Constance Kamii and Rheta DeVries, and works with other teachers to further develop the curriculum.

Mika Honda, BA, is a teacher of five-year-olds at Midori Childcare Center in Fukuyama, Japan. She works with other teachers to use constructivist activities and better ways of teaching based on Piaget’s constructivism.

Constance Kamii, PhD, is professor of early childhood education at the University of Alabama at Birmingham. She studied under Piaget on and off for 15 years and works with teachers to develop a curriculum based on his theory.
**How to Play Card Dominoes**

**Number of players:** Three is ideal. (It is possible for four children to play, but the players will be more active mentally if they do not have to wait while the fourth player takes a turn.)

**Materials:** Playing cards ace through 10 or similar homemade cards 1 through 10—a total of 40 cards. (When regular playing cards are used, it is best to white-out all the small symbols below the numerals because young children often count all the symbols, including the small ones, and say, for example, that the numeral 3 (above) says five. It is also best to white-out the As on the aces and change them to 1s.)

**The rules**
1. All the cards are dealt to the three players.
2. The players who have 1s put them down in a column in the middle of the table.
3. The children decide who will go first, second, and third.
4. Players take turns, putting down one card at a time. They make a matrix by continuing each line without skipping any number. For example, in the situation depicted above, a player can put down a red 2, a black 4, a blue 2, or a green 3.
5. Anyone who does not have a card to use must pass.
6. The first player to use up all his or her cards wins.

**Possible modifications:** Children can start the game by putting down all the 10s and proceeding in descending order, from 10 to 1. They can also begin by putting down all the 5s (or any other number) and go “up” (6, 7, 8 . . . ) and “down” (4, 3, 2, 1).

**How to Play Lining Up the 5s**

**(A Modification of Card Dominoes)**

**Number of players:** Three is ideal to maximize children’s possibility of being mentally active.

**Materials:** Three sets of 10 cards numbered 1 to 10 (a total of 30 cards) and 12 counters. Each set of cards is made with tagboard of a different color, such as pink, yellow, and blue (depicted above). (The number of cards is reduced from 40 to 30 because mental relationships are easier to make when each player gets no more than 10 cards.)

**The rules:** The rules are similar to those for Card Dominoes, except for a few differences.
1. All the cards are dealt, and the children align their cards face up in front of themselves. (This rule seems nonsensical to adults, who try to prevent others from seeing their cards. As we will show, however, young children cannot “see” things that are observable because they cannot make logico-mathematical relationships.)
2. Players who have 5s put them down in a column in the middle of the table (see Situation 1, p. 3).
3. The children decide who will go first, second, and third.
4. Players take turns, putting down one card at a time. They make a matrix by extending each line, by color, to the right or left, without skipping any number (6, 7, 8 . . . and 4, 3, 2, 1).
5. Anyone who does not have a card to use must pass. Each time a player passes, he or she takes a counter. Players can pass only three times. When a player with three counters must pass a fourth time, that player is out of the game. He or she puts down in the matrix all the cards remaining in his or her hand. In this situation it is often necessary to skip one or more numbers, leaving blank spaces in the matrix between cards that are not consecutive.
6. The first player to use up all his or her cards wins.

**Notes:** We introduced the rule of players passing no more than three times because (a) many children were passing without systematically examining all their cards for possible use and (b) the more advanced players passed just to prevent others from using their cards.

The major change from Card Dominoes is the rule about players aligning all their cards in front of themselves, face up. Being able to see everybody else’s cards encourages children to make logico-mathematical relationships, as we show in the examples described in the text.
Example 1: The children play the first card they happen to notice as being usable.

Sae (6 years and 5 months old) could have used Y4, Y6, or P6 (indicated by # signs in the diagram labeled Situation 1). She put Y4 down, not noticing that playing P6 would have been more advantageous, enabling her to use two other pink cards later—P7 and P10. Putting Y4 down enabled her to use only one card, Y2, later.

Yui (5 yrs., 10 mos.) could have used Y3, B4, or P4. She put Y3 down, not noticing that it would have been more advantageous to play B4, since she had two other blue cards that had to be used later—B3 and B2.

Daiki (6 yrs., 3 mos.) put B6 down, the only possibility he had.

Sae put Y6 down, still not noticing that playing P6 would have been more advantageous.

Yui took a long time to look at her cards and decided to pass. She took a counter. Yui did not notice B4 or P4, which she could have used.

Daiki passed, overlooking his Y7, which he could have played. He took a counter.

All three children—Sae, Yui, and Daiki—looked haphazardly for any usable card, overlooking those they could have used to their later advantage, because their logico-mathematical thinking was not developed enough to consider all of their cards systematically, one by one. Nor did it occur to them to try to prevent others from using their cards, because they did not think ahead—they thought only about their present turn. Future possibilities were totally nonexistent in their minds.

Example 2: When a child has more than one usable card, as well as other same-color cards that must be used later, he plays a card that has an immediate successor (e.g., using P7 when he has P8).

The following account focuses on Takuma (six years old) because he fits this description more consistently than Shu, and Keichi does not fit it at all. Before reviewing the children’s card playing in Example 2, however, it is necessary to explain five aspects of logico-mathematical relationships distinguished by Piaget.

Logico-mathematical knowledge consists of mental relationships—relationships originating in each person’s mind. The first aspect of such knowledge, as seen in the table “Aspects of Logico-Mathematical Knowledge,” is classification (also called categorization or sorting). Classification means mentally putting together things that are alike and separating those that are different (Inhelder & Piaget [1959] 1964). The second aspect is seriation, which means mentally ordering things according to their differences (Inhelder & Piaget [1959] 1964). The third aspect,
number (or numerical relationships), as well as the last two, spatial and temporal relationships, are self-explanatory (Piaget & Szeminska [1941]1965; Piaget [1946] 1969; Piaget & Inhelder [1948] 1960).

Takuma (6 yrs.) could have played B4, for which he did not have the immediate successor (as can be seen in Situation 2a), or P6, which could be followed by his P7, its immediate successor. He decided to use P6.

Shu (6 yrs., 9 mos.) put P4 down.
Keiichi (6 yrs., 3 mos.) put Y6 down, the only card he could use.
Takuma could have used Y7, B4, or P7 (as can be seen in Situation 2b). He decided to use Y7, which could be followed next turn by his card Y8, its immediate successor.
Shu put P3 down.
Keiichi put P2 down, the only card he could play.
Takuma (in Situation 2c) put P7 down, knowing that he had P8.
Shu put P1 down.
Keiichi passed, as he did not have anything he could play. He took a counter.

Takuma carefully looked at the cards of the players to his left and right and decided to put B4 down.

In Situation 2a Takuma made a temporal relationship between “now” and “my next turn” and coordinated it with the numerical “next-to” relationship between P6 and P7. He thus reasoned that nobody else would be helped by his use of P6 because he held P7.

By the time Takuma got to Situations 2b and 2c, it was clear that he had made a new category in his mind, consisting of Y7 and P7. These were the “perfectly safe cards to use” in view of the fact that he held Y8 and P8.

After using Y7 and P7, Takuma decided to play B4 (rather than Y8 or P8), noticing that he would have to use B2 later. He could have used Y8, but that would have opened the way for Keiichi to use Y9. He could also have played P8, but that would have allowed Shu to use P9. Takuma thus made many more relationships than the players in Example 1. He made “next-to” relationships not only with his own cards but also between his cards and other players’ cards.

Takuma illustrates an important theoretical point: development in one aspect of logico-mathematical knowledge stimulates development in other areas. Because he could think about the future, he felt the desirability of categorizing and seriating all of his cards to be better able to find the ones that were next to each other numerically and temporally. As can be seen in Situations 2a, 2b, and 2c, Takuma was the only player in Example 2 to group all of his yellow cards together, all of his blue cards together, and all of his pink cards together. Within each category, he seriated his cards numerically and spatially. This spatial arrangement facilitated the making of new categories in his mind—“cards that follow each other” (i.e., P6–P7–P8 and Y7–Y8) and “cards that will help me but not anybody else” (i.e., P6, P7, and Y7).

Example 3: When a child has more than one usable card, each with other same-color cards that must be used later, she uses first the ones that have almost-immediate successors (e.g., using Y6 first when she has Y8).

We will focus on Nami (age six-and-a-half) in the following account, because she is the only one in her group to fit this description. As seen in Situation 3a, this was a rather advanced group, since all three of the children categorized and seriated their cards.

Nami (6 yrs., 6 mos.) could have used Y6, B6, P6, or P4 in Situation 3a. She decided to use Y6 (knowing that she had Y8) rather than P6, for which she held the immediate successor, P7.
Harumi (6 yrs., 1 mo.) put B3 down, the only card she could use.
Mie (6 yrs., 3 mos.) put B2 down (knowing that she had B1).
Takuma (6 yrs., 6 mos.) put P7 down.
Yuuta (7 yrs., 2 mos.) put P8 down, not noticing the desirability of using B6 or P4 first.
Keiichi put B4 down.
Takuma put B3 down.
Yuuta passed, overlooking B6 and P4, which were both usable, and took a counter.
Keiichi put Y9 down.

Keiichi used P6 first “because 7, 8, and 9 would have to go in between [between P6 and P10],” he explained later. He used B4 next “because only 3 and 2 would have to go in between [between B4 and B1].” Keiichi thus used his knowledge of number to estimate the length of time he would have to wait after putting each card down. He used these estimates to decide which cards to use first, second, and third.

The educational value of playing Lining Up the 5s

We found that by modifying Card Dominoes, we could make it more appropriate developmentally for children. The idea of letting everybody see all the cards may seem peculiar to adults, but the preceding examples demonstrate that children can “see” only what their respective levels of logico-mathematical thinking enable them to see. In Example 1, children often did not see the cards they could have used. Although they looked at other players’ cards too, they did not get any useful information from them. Being able to look at all 30 cards all the time made it possible for the children to make increasingly higher levels of logico-mathematical relationships. In Example 2, Takuma made the temporal relationship of “next” and coordinated it with the numerical “next-to” relationship. In Example 3, Nami used the “next-to” relationship to construct the “next-to-the-next” relationship (two cards with one number missing in between). In Example 4, Keiichi made even more extensive relationships, such as “when two numbers are missing in between” and “when three numbers are missing in between.”

In the four examples, we can see that progress in one aspect of logico-mathematical knowledge stimulated progress in other areas. As they coordinated the numerical and temporal intervals, children created the category “the best cards to use first.” But the criterion of “the best cards” changed as the children progressed in their logico-mathematical development. In Example 2, the best card to use first was the one that had its immediate successor. In Example 3, it was one that had the successor of its immediate successor. In Example 4, it was the one that had the most
distant successor. Keiichi in Example 4 even seriated “the card to use first,” “the one to use second,” and “the one to use last.” His numerical reasoning too developed to the point of comparing the distance between 6 and 10 with the distance between 4 and 1.

By analyzing the videotaped play of many children over many months, we found numerous examples of the interrelated ways in which their logico-mathematical thinking developed. When Takuma in Example 2 began to think about his next turn and the number that came next, he “saw” the desirability of spatially organizing all of his cards by categorizing them and seriating them. This spatial arrangement in turn helped him and other children in Examples 3 and 4 to make more extensive numerical and temporal relationships and new categories (a category like “6 and 7” and “4 and 3”; a category like “6 and 8” and “4 and 2”; and a category like “6 and 9” and “4 and 1”). Once children made these categories, it became possible for Keiichi in Example 4 to seriate the categories into “cards that should be used first, second, and last.” In previous research too (Kamii, Miyakawa, & Kato 2004) we found that the construction of new spatial relationships led to the creation of new categories and seriational and temporal relationships.

**Thinking beyond standards**

The preceding categories that the players created are much more abstract than those children can create in sorting activities involving squares, rectangles, “red ones,” “blue ones,” and so on. The seriation involved in “cards to be used first, second, and last” is likewise much more abstract than what can be done with Montessori sticks and cylinders. If we had to set standards for mathematics in kindergarten, we would never think of including the high-level logic that we saw in Lining Up the 5s.

The joint position statement “Early Childhood Mathematics: Promoting Good Beginnings,” by NAECY and NCTM (National Council of Teachers of Mathematics) (2002), suggests the development of “well-aligned systems of appropriate high-quality standards, curriculum, and assessment” (p. 4). However, as far as objectives and standards are concerned, we can find only fragments of mathematics, such as sorting and comparing quantities, in this document.

The difficulty of conceptualizing standards for three- to six-year-olds’ mathematics education stems from the fact that subjects such as mathematics, science, and social studies are not yet differentiated in three- to six-year-olds’ thinking. For example, when young children serve coffee or go shopping in their pretend play, it is impossible to categorize this activity as mathematics, science, or social studies. When they paint, likewise, the activity inseparably involves mathematics (e.g., three brushes for three different colors), science (e.g., the drying of paint), and art.

A better guide than a subject-matter approach is the Piagetian framework that emphasizes the logico-mathematical foundation of all knowledge. The principle of teaching that flows from this conceptualization is to encourage children to think logico-mathematically because it is by thinking, or by being mentally active, that children build logico-mathematical knowledge.

One of the principles of teaching given in the NAEYC/NCTM position statement is “Integrate mathematics with other activities and other activities with mathematics” (2002, 8). An example given in that document illustrating this principle is to say, when children are lining up, that those wearing something red can get in line first, those wearing blue can get in line second, and so on. This is probably an example of encouraging children to categorize things and to put them in temporal sequence. As we saw earlier, however, logico-mathematical relationships do not develop in this kind of sterile, bit-by-bit way.

Logico-mathematical relationships are messier and more complex, and they develop in interrelated ways. Developing activities that challenge children to think logico-mathematically in their play may be a more fruitful focus of our efforts than trying to specify standards that may well be off the mark.

Play has long been valued in early childhood education, and we will do well to analyze it with depth and precision not only in card games but also in other kinds of play that naturally appeal to young children.

**References**


Copyright © 2006 by the National Association for the Education of Young Children. See Permissions and Reprints online at www.journal.naeyc.org/about/permissions.asp.